

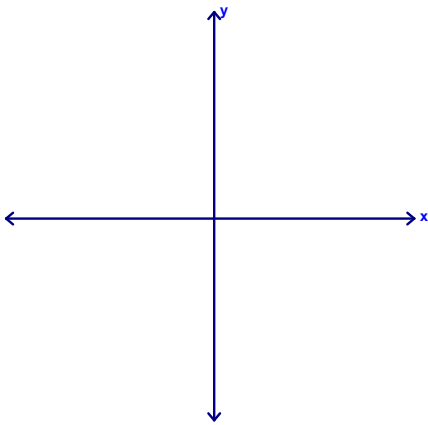
Section 2.2 – Polynomials of Higher Degree

For a graph of $y = x^n$,

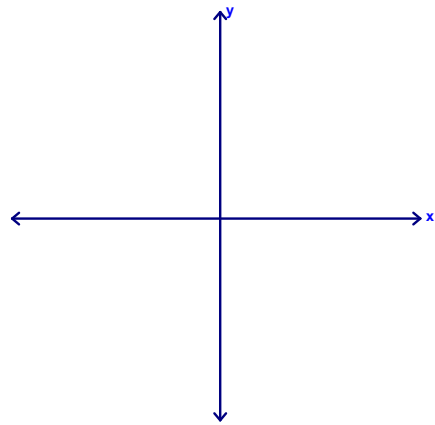
- if n is **even**, the graph touches (bounces off) the axis at the x-intercept.
- if n is **odd**, the graph crosses (passes through) the axis at the x-intercept.

The Leading Coefficient Test $\left(f(x) = a_n x^n + \dots + a_1 x^1 + a_0\right)$

When n is odd.....

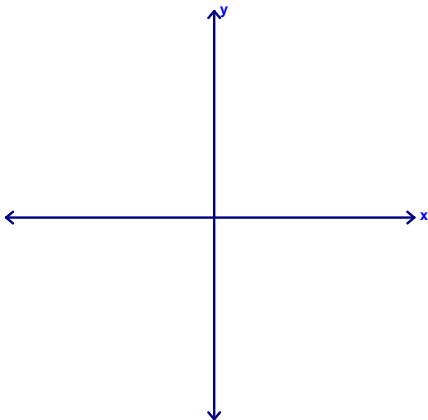


$$a_n > 0$$

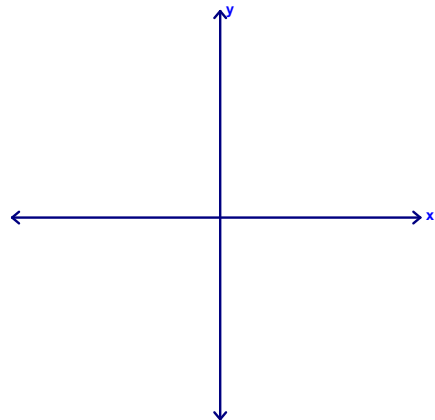


$$a_n < 0$$

When n is even.....



$$a_n > 0$$



$$a_n < 0$$

Section 2.2 – Polynomials of Higher Degree

Applying The Leading Coefficient Test

- This test is used to determine the right and left end behavior of the graph of the function.

Examples:

a) $f(x) = 2x^2 - 3x + 1$

b) $h(x) = 1 - x^6$

c) $f(x) = 2x^5 - 5x + 7.5$

d) $f(x) = -3x^7 + 2$

Zeros of Polynomial Functions

For a polynomial function f of **degree n** , the following statements are true:

- The graph of f has, at most, **$n-1$** turning points (points where the graph changes from increasing to decreasing or vice versa).
- The function f has, at most, **n** real zeros.
- If n is odd, the function f has at least one real zero.

Real Zeros of Polynomial Functions

If f is a polynomial function and a is a real number, the following statements are equivalent:

- 1) $x = a$ is a ZERO of the function f .
- 2) $x = a$ is a SOLUTION of the polynomial equation $f(x) = 0$.
- 3) $(x - a)$ is a FACTOR of the polynomial $f(x)$.
- 4) $(a, 0)$ is an X-INTERCEPT of the graph of f .

Section 2.2 – Polynomials of Higher Degree

Repeated Zeros

A factor $(x-a)^k$, $k > 1$, yields a repeated zero $x=a$ of multiplicity k .

- 1) If k is odd, the graph *crosses* the x -axis at $x=a$.
- 2) If k is even the graph *touches* the x -axis at $x=a$.

Examples: Find the zeros of a polynomial function and determine the multiplicity of each zero.

1) $f(x) = x^3 - 4x^2 + 4x$

2) $f(x) = x^4 - x^3 - 20x^2$

3) $f(x) = x^3 + x^2 - 6x$

4) $f(x) = x^3 - 6x^2 + 9x$

Section 2.2 – Polynomials of Higher Degree

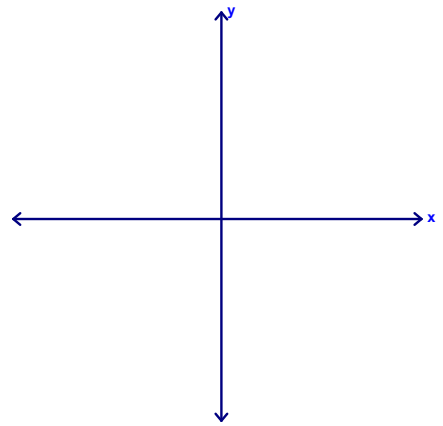
Graphs of Polynomial Functions

To sketch the graph of a polynomial function:

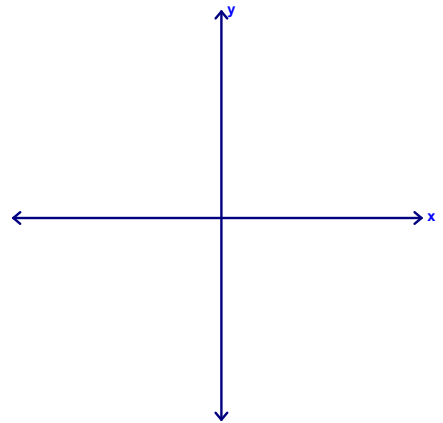
- 1) Apply the leading coefficient test.
- 2) Find the zeros of the polynomial.
- 3) Make a number line and test values between the zeros- this will determine whether your graph lies above or below the x-axis.
- 4) Draw the graph.

- 1) Sketch the graph of the polynomial functions.

a) $f(x) = x^3 - 9x$

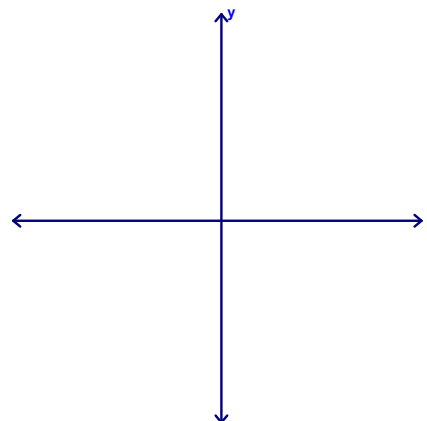


b) $-4x^3 + 4x^2 + 15x$



Section 2.2 – Polynomials of Higher Degree

c) $f(x) = -2x^3 + 12x^2 - 18x$



2) Find the equation in standard form of a polynomial of degree n that has the given zeros.

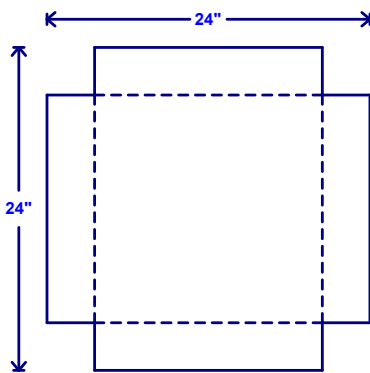
a) zeros: $x = -8, -4$
degree: $n = 2$

b) zeros: $x = 9$
degree: $n = 3$

c) zeros: $x = 2, 4 + \sqrt{5}, 4 - \sqrt{5}$
degree: $n = 4$

Section 2.2 – Polynomials of Higher Degree

- 3) An open box with locking tabs is to be made from a square piece of material 24 inches on each side, this is to be done by cutting equal squares from the corners and folding along dashed lines shown in the figure.



- a) What is the volume of the box in terms of x ?

$$V(x) =$$

- b) What is the domain of the function V ?

- c) Sketch a graph of the function and find the value of x that will give the maximum volume.



HW Day 1 ½: p. 148-149 #1-8 all, 13-21 odd, 27-30 all.

HW Day 2: p. 148-150 #9, 11, 43, 44, 47-49, 58, 59, 67, 70, 71